

# Time-Dependent Linear Response for Coupled Electron-Boson Systems

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Time-dependent linear response theory<sup>1</sup> is the most widely used approach for calculating excitation energies of electronic systems. In doing so, the Casida equation<sup>2</sup> is solved. This formalism is not restricted to electronic systems, but can also be applied to systems coupled to other fermionic particles (e.g. protons<sup>3</sup>), or to bosonic (quasi-)particles, such as photons<sup>4</sup>, Drude oscillators, or nuclear vibrations:

$$\begin{pmatrix} \mathbf{A}^I & \mathbf{A}^{I\leftrightarrow II} & \mathbf{B}^I & \mathbf{B}^{I\leftrightarrow II} \\ (\mathbf{A}^{I\leftrightarrow II})^\dagger & \mathbf{A}^{II} & (\mathbf{B}^{I\leftrightarrow II})^\dagger & \mathbf{B}^{II} \\ -(\mathbf{B}^I)^* & -(\mathbf{B}^{I\leftrightarrow II})^* & -(\mathbf{A}^I)^* & -(\mathbf{A}^{I\leftrightarrow II})^* \\ -(\mathbf{B}^{I\leftrightarrow II})^T & -(\mathbf{B}^B)^* & -(\mathbf{A}^{I\leftrightarrow II})^T & -(\mathbf{A}^{II})^* \end{pmatrix} \begin{pmatrix} \mathbf{x}^I \\ \mathbf{x}^{II} \\ \mathbf{y}^I \\ \mathbf{y}^{II} \end{pmatrix} = \hbar\omega \begin{pmatrix} \mathbf{x}^I \\ \mathbf{x}^{II} \\ \mathbf{y}^I \\ \mathbf{y}^{II} \end{pmatrix} \quad (1)$$

Hereby, the matrices and vectors are defined in a basis of single-particle excitations. The  $\mathbf{A}$  matrices and  $\mathbf{x}$  vectors contain information about occupied-virtual excitations, while the  $\mathbf{B}$  matrices and  $\mathbf{y}$  vectors contain information about virtual-occupied excitations (i.e. deexcitations). The superscript denotes whether a submatrix/subvector contains contributions from subsystem I or II or whether it is an interaction term (I  $\leftrightarrow$  II). An extension to additional subsystems is straightforward.

Starting from a mean-field treatment, the  $A$  and  $B$  matrices consist of the occupied-virtual elements of the derivative of the Fock matrix w.r.t. the density matrix. With this methodology, e.g. environment effects can be accurately captured at a low computational cost.

On the poster, theoretical aspects, details of a general implementation and sample applications using the Hartree–Fock approximation for the electrons will be shown.

## References

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